The 11th East Asian conference in Harmonic Analysis and Applications

Date: August 15 (Thur.) - 19 (Mon.), 2024
Place: 5th floor of International Science Innovation Building (No. 69), Kyoto University Access

– – –Abstracts of the talks on 15th – – –

Plenary talks: 50 min.

- Guozhen Lu, (University of Connecticut),
  Date: 15th 10:00 - 10:50, (This is an **Online** talk.)
  - Title: *Fractional GJMS operators on hyperbolic spaces and Sharp fractional Hardy-Sobolev-Maz'ya inequalities.*
  - Abstract: GJMS operators have played an important role in conformal geometry. While the GJMS operators of integer order are explicitly known on the hyperbolic space, the explicit formula of the fractional order GJMS operators have remained open. Using the scattering theory and Helgason-Fourier analysis, we will find the explicit formulas of such operators. Sharp fractional order Poincaré-Sobolev and Hardy-Maz'ya-Sobolev inequalities with best constants will be established as applications. This is joint work with Q. Yang.
- Doowon Koh, (Chungbuk National University),
  Date: 15th 16:10 - 17:00.
  - Title: *Restriction estimates over finite fields and their applications to certain combinatorial problems.*
  - Abstract: In 2004, Mochkenhaupt and Tao initially studied the Fourier restriction problems for various algebraic varieties over finite fields. The purpose of this talk is to address some connections between those problems and combinatorial problems over finite fields. First, we review the restriction problems for algebraic varieties over finite fields. Second, we introduce the following three combinatorial problems over finite fields: the Erdös-Falconer distance problem, the point-plane incidence problem, and the orthogonal projection problem over finite fields. Finally, we address how results on those problems are deduced from the finite field restriction estimates for algebraic varieties such as spheres and cones.

Talks: 30 min.

- Satoshi Masaki, (Hokkaido University),
  Date: 15th 11:10 - 11:40.
  - Title: *Small data scattering for NLS with a homogeneous nonlinearity in the weighted space.*
  - Abstract: In this talk, we consider the nonlinear Schrödinger equation with a general homogeneous nonlinearity in one and two dimensions. It is shown by Kato (1994) that, when the power of the nonlinearity is bigger than the Strauss exponent, the small data global well-posedness and scattering are established for a wide class of homogeneous nonlinearity using the exotic Strichartz estimates. On the other hand, when the degree is less than the Strauss exponent, these properties are not always guaranteed. Affirmative results are known for the case where the nonlinearity is gauge-invariant. However, GWP fails even for small data in certain classes of nonlinearity. We identify a class of nonlinearities, including non-gauge-invariant ones, for which small data GWP and scattering hold in the standard weighted space.
- Ye Zhang, (Okinawa Institute of Science and Technology),
  Date: 15th 11:10 - 11:40.
  - Title: *Loomis-Whitney inequalities on Heisenberg groups.*
  - Abstract: In this talk I will first recall some basic results about Brascamp-Lieb inequalities on Euclidean spaces. Then I will present some recent progress of nonlinear Brascamp-Lieb inequalities. Finally, by the equivalence between the Brascamp-Lieb inequality and the subadditivity of the entropy, I will deduce the Loomis-Whitney inequality on higher dimensional Heisenberg groups based on the one on the first Heisenberg group.

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- Kiyeon Lee, (KAIST),
  Date: 15th 12:00 - 12:30.
  - Title: *The global dynamics for the Maxwell-Dirac system.*
  - Abstract: In this talk, we study the $(1+3)$ dimensional massive Maxwell-Dirac system in the context of global existence and asymptotic behavior of solutions under the Lorenz gauge condition, as well as the modified and linear scattering phenomena for the Dirac spinor and the electromagnetic potential, respectively. The primary ingredients of this talk are a vector fields energy method combined with a detailed analysis of the space-time resonance argument. This approach allows us to establish decay estimates and energy bounds crucial for proving the main theorems. Especially, we provide the explicit phase correction arising from the strong nonlinear resonances.
- Takeshi Kawazoe, (Keio University),
  Date: 15th 12:00 - 12:30.
  - Title: *On Hardy spaces on Riemannian symmetric spaces of rank one.*
  - Abstract: Let G/K be a Riemannian symmetric space and K/M the boundary of G/K. As an analogue of the classical case, for $h \in L^p(K/M)$, $1 \leq p < \infty$, the Poisson transform $P_\lambda h(ka)$ has a boundary value and $\lim_{a\to\infty} \phi_\lambda(a)^{-1} P_\lambda h(ka) = h(k)$. The

Hardy space $\mathcal{H}^p_\lambda(G/K)$ can be defined as the space of analytic functions on G/K satisfying $\Delta H = \chi(\Delta)H$ and

$$\sup_{a\in A} \phi_{\Re\lambda}(a)^{-1}\Big(\int_K |H(ka)|^p dk\Big)^{\frac{1}{p}} < \infty.$$

On K/M, $\mathcal{H}^p_\lambda(K/M)$ is defined as the space of the boundary values (see [1], [2], [3]). On the other hand we can also define a real Hardy space on $H^p(G/K)$ by using a radial maximal operator. Originally, these spaces are independent objects and have no known relationships. In this talk, let $G = SU(1,1)$; $G/K = D = \{|z| < 1\}$ and $K/M = \partial D = \mathbb{T}$. We shall consider a modified Poisson transform $\mathcal{P}_\lambda h(ka) = \phi_\lambda(a^*)^{-1} P_\lambda h(ka^*)$ for $h \in \mathcal{H}^1_\lambda(K/M)$. Then we show that for $F \in H^1(G/K)$,

$$\|F *_{K/M} \mathcal{P}_\lambda h\|_{H^1(G/K)} \le c\|F\|_{H^1(G/K)}\|h\|_{\mathcal{H}^1_\lambda(K/M)}.$$

[1] E. P. van den Ban and H. Schlichtkrull, Asymptotic expansions and boundary values of eigenfunctions on a Riemannian symmetric spaces, J. Reine Angrew. Math. 380 (1987), 108-165.

[2] H. L. Michelson, Fatou theorems for eigenfunctions of the invariant differential operators on symmetric spaces, Hans. Amer. Math. Soc. 177 (1973), 257-274.

[3] Salem Ben Saïd, T. Oshima, N. Shimeno, Fatou's theorems and Hardy-type spaces for eigenfunctions of the invariant differential operators on symmetric spaces, Intern. Math. Research Notice, 16 (2003), 915-931.

– – –Lunch break – ––

- Naoya Hatano, (Chuo University),
  Date: 15th 14:30 - 15:00.
  - Title: *Boundedness of composition operators from Lorentz spaces to Orlicz spaces.*
  - Abstract: The boundedness (continuity) of composition operators from some function space to another one is significant, though there are few results about this problem. Thus, in this talk, we introduce necessary and sufficient conditions on the boundedness of composition operators from Lorentz spaces to Orlicz spaces. We emphasize that the measure spaces associated with the Lorentz space may be different from those associated with the Orlicz spaces. Moreover we give some examples and counterexamples of the composed mappings in the conditions.
- Jun Cao, (Zhejiang University of Technology),
  Date: 15th 14:30 - 15:00.
  - Title: *On the relative-boundedness of Schrödinger operators.*
  - Abstract: Relative boundedness is a fundamental property appearing in the study of Schrödinger operators. From the perspective of perturbation theory, it can extend many desirable properties of the Laplace operator (such as self-adjointness and spectral information) to Schrödinger operators, by assuming the potential engergy sufficiently small. In this report, we motivate the study of relative boundedness from the heat kernel estimate for Schrödinger operators. We primarily focus on three types of relative boundedness and their respective characterizations. Additionally, we discuss how these characterizations are connected to harmonic analysis tools such as capacity, Carleson measures, sparse control, and inverse Hölder classes.

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- Lu Chen, (Beijing Institute of Technology),
  Date: 15th 15:20 - 15:50.
  - Title: *Some new progress on quantitative stability of geometric inequality.*
  - Abstract: In this talk, we will first introduce the sharp geometric inequality and their stability. Then we present recent progress on the quantitative stability for Hardy-Littlewood-Sobolev (HLS) inequality, fractional Sobolev inequality and trace Sobolev inequality. Finally, we also will discuss the optimal asymptotic lower bound for stability of HLS , fractional Sobolev inequality and Log Sobolev inequality on the sphere. This talk is based on the joint work with Prof. Lu from Connecticut University and Prof. Tang from Beijing Normal University.
- In-Jee Jeong, (Seoul National University),
  Date: 15th 15:20 - 15:50.
  - Title: *Degenerate dispersive equations and wave packets.*
  - Abstract: Degenerate dispersive PDEs appear in various areas of physics. Unlike their non-degenerate counterparts, wellposedness for those is a very subtle issue. This issue can be understood in terms of high frequency wave packets, which are roughly speaking highly oscillatory Gaussian-like approximate solutions. We explain main ideas that go into the construction of wavepackets, and how to interpret consequences for wellposedness.